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Analysis of the One-Level Sealed Bidding on Effectiveness under Different Bidding Variables

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Abstract: The one-level sealed bidding is studied under the circumstance of project construction in build-operate-transfer (BOT) mode, where the bidding variable is considered separately as concession term, toll per product, payment and the total revenue. A game model with incomplete information among tenderers is presented and the optimal bidding strategies of tenderers are given. When the tenderer's construction cost belongs to uniform distribution, the optimal bidding strategy is related to his cost, the numbers of all tenderers, his relative cost coefficient, demand in unit time and so on. Furthermore, when the anticipations of market demand among the tenderers are different, the optimal bidding strategies under the above bidding variables may fail; when the operation and maintenance cost could be ignored, the bidding variable of total revenue is the best choice.

Keywords: bidding strategy, bidding variable, BOT, the one-level sealed bidding

1 Introduction

Build-Operate-Transfer (BOT) is a popular form of franchising in which a concession contract is signed between the private firm (the project company) and the government. Under this contract, the firm takes on the task of building and financing the costs of a project. As compensation, the firm has the right to charge tolls to users during a fixed period. This kind of financing is a good measure to ease up the supply of infrastructure and the government could benefit from the private firm for its advance technology and management[1]. For a project, there used to be several firms which are different at efficiencies (for example, the construction cost is different) to compete. A good method to reveal the private information of the agents and decrease the agent fee is auction[2-3], which is widely used in economic field. It could also be used to solve the competition among the firms under BOT. The one-level sealed bidding is a type in which each tenderer separately writes down his own bid （or ask）and seals it in an envelope. The tender opens these envelopes and the tenderer who bids highest or asks lowest can get the object. In this case, each tenderer chooses bidding strategy according to his and other tenderers' evaluation of the object. What the auction winner pays is the difference between his evaluation and the execution price. Myerson[4] and Riley[5] do some creative work on the mechanism design and the equilibrium bidding strategies. Assuming the cost of tenderers are independent and submit to the [0,1] uniform distribution, Zhang[6] points out the tenderer's optimal strategy depends on his own cost and the number of tenderers; Committee[7] and Yan[8] draw similar conclusions. Under the same assumptions as the above mentioned documents, Zhang[9] suggests that the optimal strategy is additionally related to the cost distribution span of tenderers. Xiao[10] applies the one-level sealed bidding to analyze the initial emissions permits. Wang[11] discusses one-level sealed bidding under one-pay and all-pay mechanisms then analyzes the application of its conclusions to reach and development. The bidding variables of one-level sealed bidding in [6-11] are humdrum and almost directly related to the tenderers' construction cost. Considering the aspect of the application of the game theory in BOT, Yang[12] and Zhou[13] establish the model of complete information and sequent actions between the government and the project company, while the selection of agent for the government that is the competition among the firms is not discussed. As the auction in BOT, the problem originates from the franchise bidding theory of Demsetz(1968)[14]. The common bidding variables are concession term, toll of per product, payment and total revenue, which are all indirectly related to the tenderers' construction cost and differ from the situation in usual engineering auction. Beato [15] and Guasch [16] demenstrate some case study on bidding variables; Tirole[17] discusses flexible concession term; Engel[18-19] further propose least-present-value -of-revenue auction, but the operation and maintenance cost is out of consideration.

In this paper, the game models of competition among the tenderers are established under the mentioned bidding variables in BOT. The optimal bidding strategies for the tenderers are given while their cost distribution is unknown, and the effectiveness of the strategies is analyzed. At last, suggestions in application are proposed for the tender (the government).

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2 Model

It bases on the assumptions that:

(1)The object is a single project;

(2)The number of the eligible agents is n and they are all risk neutral and rational;

(3)When the construction of the project is up to scratch, the cost of the tenderer $i(i=1,2,\cdots,n)$ is c_i and that is private information.

(4)The tenderer $i(i=1,2,\cdots,n)$ has the information that the construction costs of the other $n-1$ tenderers are independent and belong to the same distribution during $[\underline{C},\bar{C}]$, the density function $f(c)$, the distribution function $F(c)$ and $F(\underline{C})=0, F(\bar{C})=1$.

The mechanism is one-level sealed bidding, the game process among the tenderers has the characteristics of incomplete information and static. Under the specific bidding variable, the decision-making of the tenderer bases on his own cost and the Bayesian equilibrium bidding strategy is $B_i(c_i), c_i \in [\underline{C},\bar{C}]$. Assuming that $B_i(\square)$ is a critical-monotonic-differential function, it is according with the practice because the bidding decision-making bases on the cost. For the symmetrical game among the tenderers, we have

$$B_1(\square)=B_2(\square)=\cdots=B_n(\square)=B(\square)$$

2.1 Analysis under the bidding variable of concession term

The bidding variable is concession term T. The concession term includes the instruction time, the operation time and so on. Here the instruction time is assumed to be the same to all tenderers, so the concession term means the total time the tenderers could benefit from the project.

The toll per product P is made by the government to the market and the social welfare. The market demand in unit time Q is variable to P and is assumed to be common knowledge for convenience of analysis. The operation and maintenance cost in unit time M is assumed to equal for all tenderers, which is based on the premise that the construction of the project is up to scratch.

The cost of tenderer i is marked as c. When it could not make confusion, c is the briefness of c_i. The bidding strategy of the tenderer i is $B_i(c)=t, t\in T$ (T is a set of all concession terms proposed by the tenderers) and the bidding decision-making of concession term t is based on the tenderer's cost c. Here $B_i(\square)$ is critical increasing, in another word, the lower the cost, the lower the concession term.

When tenderer i chooses t as the concession term the probability he will be the winner is as follows (the decision-making of all tenderers are independent):

$$\begin{aligned} prob_i(t) &= [prob(t<t_j)]^{n-1} \\ &= [prob(B^{-1}(t)<B^{-1}(t_j))]^{n-1} \\ &= [1-F(B^{-1}(t))]^{n-1} \end{aligned}$$

The expected interest of tenderer i is

$$\Pi_i^e(t)=((PQ-M)t-c)[1-F(B^{-1}(t))]^{n-1}$$

The first-order maxim condition is

$$(PQ-M)[1-F(B^{-1}(t))]^{n-1}-((PQ-M)t-c)$$
$$*(n-1)[1-F(B^{-1}(t))]^{n-2}\frac{dF}{dB^{-1}(t)}\frac{dB^{-1}(t)}{dt}=0$$

Using $t=B(c)$ and rewriting this equation

$$(PQ-M)(1-F(c))\frac{dB}{dc}-((PQ-M)B(c))(n-1)\frac{dF}{dc}$$
$$=-c(n-1)\frac{dF}{dc}$$

that is

$$d((PQ-M)B(c))(1-F(c))^{n-1}=cd(1-F(c))^{n-1}$$

By integrating both sides from c to $\bar{C}$, and using $F(\bar{C})=1$, the equilibrium bidding strategy of tenderer i is as follows:

$$B(c)=(c+\frac{\int_c^{\bar{C}}(1-F(s))^{n-1}ds}{(1-F(c))^{n-1}})(PQ-M)^{-1}$$

When $F(c)$ follows uniform distribution, the optimal strategy of the tenderer i is:

$$\begin{aligned} B(c) &= (\frac{\bar{C}}{n}+\frac{n-1}{n}c)(PQ-M)^{-1} \\ &= c(\frac{\varphi_i}{n}+\frac{n-1}{n})(PQ-M)^{-1} \end{aligned} \quad (1)$$

where $\varphi_i=\frac{\bar{C}}{c}(\varphi_i \ge 1)$ is the relative cost coefficient of the tenderer i (Zhang[9] defines the concept of the cost scattered span, in this paper, it is extended to relative cost coefficient and suitable for any tenderer i). Obviously, formulation (1) is a critical increasing function of cost and satisfies "the lower the cost, the lower the concession term", so the winner tenderer is the one who has the biggest relative cost coefficient (who has the lowest cost) and the optimal bidding strategy is effective to distinguish the excellent tenderer under the above assumptions.

From formulation (1), the optimal bidding strategy not only depends on the tenderer's cost, the number of tenderers and the tenderer's relative cost coefficient, but also depends on the toll per product, the demand in unit time, the operation and maintenance cost in unit time.

Now relax the assumption of Q. If the anticipations of market demand among the tenderers are different, the formulation (1) could not assure the tenderer who has the lowest cost to be the winner. The analysis is as follows.

Marking the cost of tenderer 1 as $c_1=\underline{C}$ and the anticipation of the market demand as Q_1; marking the cost of tenderer $j(j \neq 1)$ as c_j and the anticipation of the market demand as Q_j.

When $Q_j = \lambda Q_1 (\lambda > 1)$, if $B(c_1) > B(c_j)$, so:

$$(\frac{\bar{C}}{n} + \frac{n-1}{n} c_1)(PQ_1 - M)^{-1} > (\frac{\bar{C}}{n} + \frac{n-1}{n} c_j)(P\lambda Q_1 - M)^{-1}$$

And the solution is as follows:

$$\lambda > 1 + \frac{(c_j - c_1)(n-1)}{\bar{C} + (n-1)c_1}(1 - \frac{M}{PQ_1}) \qquad (2)$$

In formulation (2), it is obvious that $c_j > c_1$ due to tenderer 1 is the one who has the lowest cost; it is necessary for the tenderer 1 to participate in the auction that the revenue in unit time PQ_1 could compensate the operation and maintenance cost in unit time, that is $PQ_1 > M$. So $\frac{M}{PQ_1} < 1$ comes into existence. Then we have the conclusion that the λ satisfying the formulation (2) could make the tenderer j who is not the excellent to be the winner and then the above bidding strategy is not effective to distinguish the tenderers.

2.2 Analysis under the bidding variable of toll per product

The bidding variable is P which means the toll per product.

The market demand in unit time Q is variable to P. The demand curve $Q(p)$ is assumed to be common knowledge for convenience of analysis. The operation and maintenance cost in unit time M is assumed to equal for all tenderers, which is based on the premise that the construction of the project is up to scratch.

The concession term includes the instruction time, the operation time and so on. Here the instruction time is ignored as it is assumed in section 2.1, so the concession term T means the total time the tenderer could benefit from the project and it is made by the government to the market and the social welfare. For the determination of the concession term, the discussion in Li[20] could also be used for reference.

The bidding strategy of the tenderer i is $B_i(c) = p, p \in P$ (P is a set of all tolls per product proposed by the tenderers) and the bidding decision-making of toll per product p is based on the tenderer's cost c. Here $B_i(\cdot)$ is critical increasing, in another word, the lower the cost, the lower the toll.

When tenderer i chooses p as the toll per product, the probability he will be the winner is as follows (the decision-making of all tenderers are independent):

$$\begin{aligned} prob_i(p) &= [prob(p < p_j)]^{n-1} \\ &= [prob(B^{-1}(p) < B^{-1}(p_j))]^{n-1} \\ &= [1 - F(B^{-1}(p))]^{n-1} \end{aligned}$$

The expected interest of tenderer i is:

$$\Pi_i^e(p) = ((pQ(p) - M)T - c)[1 - F(B^{-1}(p))]^{n-1}$$

The first-order maxim condition is:

$$(QT + pT\frac{dQ}{dp})[1 - F(B^{-1}(p))]^{n-1} - ((pQ - M)T - c)$$

$$*(n-1)[1 - F(B^{-1}(p))]^{n-2} \frac{dF}{dB^{-1}(p)} \frac{dB^{-1}(p)}{dp} = 0$$

and because $F(\bar{C}) = 1$, the solution is as follows:

$$pQ(p) = (c + \frac{\int_c^{\bar{C}} [1 - F(s)]^{n-1} ds}{[1 - F(c)]^{n-1}})T^{-1} + M$$

When $F(c)$ follows uniform distribution, the optimal bidding strategy of the tenderer i meets:

$$pQ(p) = (\frac{\varphi_i}{n} + \frac{n-1}{n})cT^{-1} + M \qquad (3)$$

From formulation (3), it is easy to see that under the bidding variable of the toll per product, the optimal bidding strategy not only depends on the tenderer's cost, the number of all tenderers and the tenderer's relative cost coefficient, but also depends on the concession term, the demand in unit time, the operation and maintenance cost in unit time.

If the optimal bidding strategy is effective to distinguish the excellent tenderer, formulation (3) should satisfy that price is critical increasing with cost. So the elasticity of the revenue in unit time $pQ(p)$ to price p should be positive which is based on the assumptions that the anticipations of the market demand are the same:

$$Q(p) + p\frac{dQ}{dp} > 0$$

Also, if the anticipations of market demand among the tenderers are different, the formulation (3) could not assure the tenderer who has the lowest cost to be the winner.

2.3 Analysis under the bidding variable of payment

The toll per product P is made by the government to the market and the social welfare. The market demand in unit time Q is variable to P and is assumed to be common knowledge for convenience of analysis. The operation and maintenance cost in unit time M is assumed to equal for all tenderers, which is based on the premise that the construction of the project is up to scratch. As the concession term, we also do not count the influence of instruction time as it is assumed in section 2.1, so the concession term T means the total time the tenderer could benefit from the project and it is made by the government to the market and the social welfare. The discussion in Li[20] could also be used for the determination of the concession term.

Under this kind of bidding variable, the winner of the contract should pay some money to the government at one stroke, so the tenderer who bids the highest price will be the winner. The bidding variable is m which means the payment.

The bidding strategy of the tenderer i is $B_i(c) = m, m \in Mp$ (Mp is a set of all payment-prices proposed by the tenderers) and the bidding decision-making of payment m is based on the tenderer's cost c. Here $B_i(\cdot)$ is critical decreasing, in another word, the lower the cost, the higher the payment.

When tenderer i chooses m as the payment the probability he will be the winner is as follows (the decision-making of all tenderers are independent):

$$\begin{aligned} prob_i(m) &= [prob(m > m_j)]^{n-1} \\ &= [prob(B^{-1}(m) < B^{-1}(m_j))]^{n-1} \\ &= [1 - F(B^{-1}(m))]^{n-1} \end{aligned}$$

The expected interest of tenderer i is :

$$\prod_i^e(m) = ((PQ - M)T - c - m)[1 - F(B^{-1}(m))]^{n-1}$$

The first-order maxim condition is:

$$(-1)[1-F(B^{-1}(m))]^{n-1} - ((PQ-M)T-c-m)$$

$$*(n-1)[1-F(B^{-1}(m))]^{n-2}\frac{dF}{dB^{-1}(m)}\frac{dB^{-1}(m)}{dm} = 0$$

and because $F(\bar{C}) = 1$, the solution is as follows:

$$B(c) = (PQ - M)T - (c + \frac{\int_c^{\bar{C}} [1 - F(s)]^{n-1} ds}{[1 - F(c)]^{n-1}})$$

When $F(c)$ follows uniform distribution , the optimal strategy of the tenderer i is:

$$B(c) = (PQ - M)T - c(\frac{\varphi_i}{n} + \frac{n-1}{n}) \quad (4)$$

From formulation (4), the optimal bidding strategy not only depends on the tenderer's cost, the number of all tenderers and the tenderer's relative cost coefficient, but also depends on the concession term, toll per product, the demand in unit time and the operation and maintenance cost in unit time. Obviously, formulation (4) is a critical decreasing function of cost, so the optimal bidding strategy is effective to distinguish the excellent tenderer under the above assumptions.

Similar, if the anticipations of market demand among the tenderers are different, formulation (4) can not assure the tenderer who has the lowest cost to be the winner. The analysis is as follows.

Relax the assumption of Q. Marking the cost of tenderer 1 as $c_1 = \underline{C}$ and the anticipation of the market demand as Q_1; marking the cost of tenderer $j (j \neq 1)$ as c_j and the anticipation of the market demand as Q_j and $Q_j \neq Q_1$.

When $Q_j = \lambda Q_1 (\lambda > 1)$, if $B(c_1) < B(c_j)$, so:

$$(PQ_1 - M)T - (\frac{\bar{C}}{n} + \frac{n-1}{n}c_1) < (P\lambda Q_1 - M)T - (\frac{\bar{C}}{n} + \frac{n-1}{n}c_j)$$

And the solution is as follows:

$$\lambda > 1 + \frac{1}{PQ_1T}\frac{n-1}{n}(c_j - c_1) \quad (5)$$

In formulation (5), it is obvious that $c_j > c_1$ due to tenderer 1 is the one who has the lowest cost. Then we have the conclusion that the λ satisfying the formulation (5) could make the tenderer j who is not the excellent to be the winner and then the above bidding strategy is not effective to distinguish the tenderers.

2.4 Analysis under the bidding variable of total revenue

The toll per product P is made by the government to the market and the social welfare. The market demand in unit time Q is variable to P and is assumed to be common knowledge for convenience of analysis. The operation and maintenance cost in unit time M is assumed to equal for all tenderers, which is based on the premise that the construction of the project is up to scratch.

The bidding variable L is total revenue and it means all of the income the tenderer benefits from financing the project during the concession term (to make the model concise, the time value of the cash flow is not considered).

The concession term T means the total time the tenderer could benefit from the project as it is assumed in section 2.1, so it satisfies that:

$$T = \frac{L}{PQ}$$

So the bidding variable of total revenue makes a flexible-term mechanism. When the market demand is optimistic, the concession term is correspondingly shortened; otherwise it is lengthened.

The bidding strategy of the tenderer i is $B_i(c) = l, l \in L$ (L is a set of all total revenues proposed by the tenderers) and the bidding decision-making of total revenue l is based on the tenderer's cost c. Here $B_i(\cdot)$ is critical increasing, in another word, the lower the cost, the lower the total revenue.

When tenderer i chooses l as the total revenue the probability he will be the winner is as follows (the decision-making of all tenderers are independent):

$$\begin{aligned} prob_i(l) &= [prob(l < l_j)]^{n-1} \\ &= [prob(B^{-1}(l) < B^{-1}(l_j))]^{n-1} \\ &= [1 - F(B^{-1}(l))]^{n-1} \end{aligned}$$

The expected interest of tenderer i is :

$$\prod_i^e(l) = (l - M\frac{l}{PQ} - c)[1 - F(B^{-1}(l))]^{n-1}$$

The first-order maxim condition is:

$$(1-\frac{M}{PQ})[1-F(B^{-1}(l))]^{n-1}-(l-M\frac{l}{PQ}-c)$$

$$*(n-1)[1-F(B^{-1}(l))]^{n-2}\frac{dF}{dB^{-1}(l)}\frac{dB^{-1}(l)}{dl}=0$$

and because $F(\bar{C})=1$, the solution is as follows:

$$B(c)=(c+\frac{\int_c^{\bar{C}}[1-F(s)]^{n-1}ds}{[1-F(c)]^{n-1}})(1-\frac{M}{PQ})^{-1}$$

When $F(c)$ follows uniform distribution, the optimal strategy of the tenderer i is:

$$B(c)=c(\frac{\varphi_i}{n}+\frac{n-1}{n})(1-\frac{M}{PQ})^{-1} \tag{6}$$

From formulation (6), the optimal bidding strategy not only depends on the tenderer's cost, the number of all tenderers and the tenderer's relative cost coefficient, but also depends on the toll per product, the demand in unit time, the operation and maintenance cost in unit time. Obviously, formulation (6) is a critical increasing function of cost and satisfies "the lower the cost, the lower the total revenue", so the optimal bidding strategy is effective to distinguish the excellent tenderer under the above assumptions.

Similar, if the anticipations of market demand among the tenderers are different, formulation (6) can not assure the tenderer who has the lowest cost to be the winner.

At the same time, it is important to notice that when the operation and maintenance cost could be ignored as in formulation (7), the optimal bidding strategy will be effective and have no relationship with the anticipations of market demand.

$$B(c)=c(\frac{\varphi_i}{n}+\frac{n-1}{n}) \tag{7}$$

So using total revenue as the bidding variable and peeling the operation and maintenance cost off the tenderer (for example, it could be born by the government, which is based on the premise that the construction of the project is up to scratch) is a good way to amend the auction mechanism in BOT.

3 Conclusion and suggestion

In this paper, the game models which have the characteristics of incomplete information and static among the tenderers of infrastructure BOT project are set up under the following auction variables: concession term, toll per product, payment and total revenue. The optimal bidding strategies for tenderers are given and the probability of invalidation in application is analyzed.

The suggestions for the tender (the government) are as follows. First, no matter which bidding variable is used, the government should organize the work of demand forecast and proclaim the results to the public as the reference and basement of the tenderers' bidding, due to the market demand of the project is a very important factor, then the auction will be effective and the efficient tenderer will be outstanding. From the other side, the forecast work and the proclamation will be a guard for some blind tenderer (bad efficient one) and make him to evaluate the project properly and shrink back from difficulties, so the project's risk of failing due to the defective auction mechanism could be reduced to the most extent. At last, the forms of the one-level sealed bidding are various in infrastructure BOT project and may all be invalid, so some technique is necessary to amend the mechanism(for example, to peel off the operation and maintenance cost in this paper). Additionally, the analysis of extent of failing to different bidding variables needs further consideration.

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